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MHD Effects And Heat Transfer On A Boundary Layer Flow Past A Stretching Plate With Heat Source/Sink In The Presence Of Viscous Dissipation

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Abstract
The boundary layer flow of incompressible electrically conducting fluid past a stretching plate in the presence of transverse magnetic field with heat transfer taking into the account the viscous dissipation effects is considered in this present study. The non-linear momentum boundary layer and heat transfer equations are converted into non-linear ordinary differential equations using similarity transformations. The resulting boundary value problem is solved numerically by an implicit finite difference scheme. The solution is found to be dependent on magnetic field parameter M, Source term S, Prandtl number Pr and Eckert number E_c . The results are shown graphically to illustrate the effects of these parameters on the fluid velocity and temperature distribution in the boundary layer.
Keywords: Magnetic field effects, boundary layer flow, heat transfer, stretching plate and Viscous Dissipation.

I. INTRODUCTION

The study of flow over a stretching sheet has generated much interest in recent years due to its important contribution especially in many engineering processes and industries. The applications in industries involved such as the aerodynamic extrusion of plastic sheets, glass-fiber production, condensation process of metallic plate in a cooling bath and glass and also in polymer industries. Crane [1] was the first who reported the analytical solution for the laminar boundary layer flow past a stretching sheet. After this pioneering work, the study of fluid flow over a stretching sheet has received wide attention among researchers. Gupta and Gupta [2] added new dimension to the study with suction and injection.

In recent years, MHD flow problems have become more important industrially. Indeed, MHD laminar boundary layer behaviour over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. In his pioneering work, Sakiadis [3] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid.

Further investigations on boundary layer flow and heat transfer of viscous fluids over a flat sheet are very important for development in many manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass-fiber, metal extrusion, and metal spinning. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products (Magyari & Keller [4]). Among these studies, Sakiadis [5] initiated the study of the boundary layer flow over a stretched surface moving with a constant velocity and formulated a boundary-layer equation for two-dimensional and axisymmetric flows.

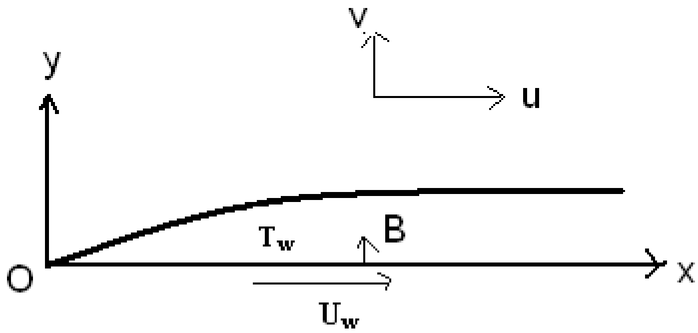
Bujurke et. al. [6] made an investigation on the heat transfer analysis in a second order fluid flow past a stretching surface with heat transfer. Datta et. al. [7] have studied the distribution of temperature in a continuous stretching sheet with uniform wall heat flux. Further flow and heat transfer from a linearly stretching sheet gained more importance due to practical applications in industrial processes. Abel and Veena [8] have analyzed visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet.

There are several other researchers who investigated various aspects of heat transfer characteristics over linearly stretching sheets Dandapat and Gupta [9], Rollins and Vajravelu [10], Lawrence and Rao [11] and Char [12]. Khan and Sanjayanand [13] studied viscoelastic boundary layer fluid flow over a quadratically stretched boundary sheet. However, heat transfer analysis was excluded from their study. Elbashbeshy [14] examined the flow and heat transfer characteristics by considering exponentially stretching sheet. Gebhart [15] has shown that the viscous dissipation effect plays an important role in natural convection in various devices processes on large scales(or large planets). Also, he pointed out that when the temperature is small or when the gravitational field is of high intensity, viscous dissipation heat should be taken into account. Therefore, the effect of viscous dissipation is more predominant in vigorous natural convection processes.

MHD flow and heat transfer of a non-Newtonian power-law fluid past a stretching sheet with suction/injection and viscous dissipation was studied by Kishan and Shashidar [16] . Kishan and kavitha [17] studied the MHD heat transfer to non-Newtonian power-law fluids over a wedge with heat source/sink in the presence of viscous dissipation. Recently Anuj kumar and Manoj kumar [18] has studied MHD boundary layer flow past a stretching plate with heat transfer. The aim of the present study is to investigate the MHD and heat transfer effects of a laminar electrically conducting fluid as a boundary layer flow over a stretching plate with heat source/sink in the presence of viscous dissipation.

II. MATHEMATICAL FORMULATION

Let us consider two dimensional laminar boundary layer flows over a stretching plate in an incompressible electrically conducting fluid, where the x-axis is along the stretching plate and y-axis perpendicular to it, the applied magnetic field B_0 is transversely to x-axis. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. Under the usual boundary layer approximations, the governing equations of continuity, momentum and energy under the influence of externally imposed transverse magnetic field are:



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{--------------------------------(1)}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta_0^2}{\rho}u \quad \text{----------------------------(2)}$$

$$v\frac{\partial T}{\partial y}=\frac{v}{p_r}\frac{\partial^2 T}{\partial y^2}+\frac{(T_p-T_\infty)h^2}{c_p}\left(\frac{\partial u}{\partial x}\right)^2+\frac{Q(T-T_\infty)}{\rho\, c_p} \quad \text{--------------------------------(3)}$$

Along with the boundary conditions :

$$y=0:\ u=x, v=0,\ T=Tp\ m>0\ \ y\to\infty:\ u=0\ ,\ T=T\infty \quad \text{--------------------(4)}$$

Here since temperature field varies with regard to y only, so $\frac{\partial t}{\partial x}=0$. Also we introduce the following non-dimensional quantities.

$$\bar{x}=\frac{x}{h},\bar{y}=\frac{y}{h},\bar{u}=\frac{uh}{v},\bar{v}=\frac{vh}{v},\theta(y)=\frac{T-T_\infty}{T_p-T_\infty} \quad \text{-------------------(5)}$$

Where Tp is the plate temperature and $T\infty$ is the temperature of surrounding. Substituting equation (5) in to (1) to (3) , these equations are reduced to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{----------------------(6)}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-Mu \qquad \text{---------------------(7)}$$

Where $M=\frac{\sigma\beta_0^2}{\rho}\frac{h^2}{v}$(non-dimensional magnetic parameter)

$$\frac{\partial^2\theta}{\partial y^2}+f(y)p_r\frac{\partial\theta}{\partial y}+\frac{v}{c_p}p_r\left(\frac{\partial u}{\partial y}\right)^2+p_r\frac{1}{v}\frac{Q\theta h^2}{\rho C_p}=0 \qquad \text{------------------------(8)}$$

Where bar has been dropped for our convenience
Along with the boundary conditions:

$y=0; u=x, v=0, \theta=1$

$y\to\infty: u=0, \theta=0$ ----------------------(9)

III. Method of Solution

We shall further transform equations (7) & (8) into a set of partial differential equations amenable to a numerical solution. For this purpose we introduce the similarity solution of the form

$$u=xf'(y) \qquad \text{---------------------(10)}$$

Also using the continuity equation (6) with equation (10) we have

$v=-\{f(y)-f(0)\}$ ----------------------(11)

Using (10) and (11) equations (7) and (8) becomes

$$f'^2(y)-f(y)f''(y)=f'''(y)-Mxf'(y) \qquad \text{----------------------(12)}$$

$$\theta''(y)+p_r f(y)\theta'(y)+P_rE_c(f''(y))^2+PrS\theta=0 \qquad \text{----------------------(13)}$$

Where $S=\frac{Qh^2}{\rho v C_p}$

Along with the boundary conditions :

$y=0: f'=1, \theta=1$

$y\to\infty: f'=0, \theta=0$ ---------------------(14)

Where we take f(0) =0, without any loss of generality.

To solve the system of transformed governing equations (12) and (13) with the boundary conditions (14), first equation (12) is linearized using the Quasi linearization technique[19]. Then equation (12) is changed to

$$[2F'f'-(F')^2-Ff''-fF''-FF'']=f'''(y)-Mxf'(y) \qquad \text{--------------(15)}$$

Where F is assumed to be a known function and the above equation can be rewritten as

$$A_0f'''+A_1f''-A_2f'+A_3f+A_4=0 \qquad \text{-----------------(16)}$$

Where
$A_0=1$
$A_1=F$
$A_2=2F'+Mx$
$A_3=F''$
$A_4=(F')^2-FF''$

Now equation 13 can be expressed in simplified form as

$$C_0\theta'' + C_1\theta' + C_2 + C_3\theta = 0 \qquad \text{--------------------- (17)}$$

Using implicit finite difference formulae, the equations (16) and (17) are transformed to

$B_0f(i+2)+B_1f(i+1)+B_2f(i)+B_3f(i-1)=B_4$ ----------------(18)
$D_0g(i+1)+D_1g(i)+D_2g(i-1)+ D_3=0$ -----------------(19)

here 'h' represents the mesh size in η direction. The system of equations (18) & (19) are solved under the boundary conditions (14) by Gauss-Seidel iteration method and computations were carried out by using C programming. The numerical solutions of f are considered as $(n+1)^{th}$ order iterative solutions and F are the n^{th} order iterative solutions. After each cycle of iteration the convergence check is performed, and the process is terminated when $|F - f| < 10^{-4}$.

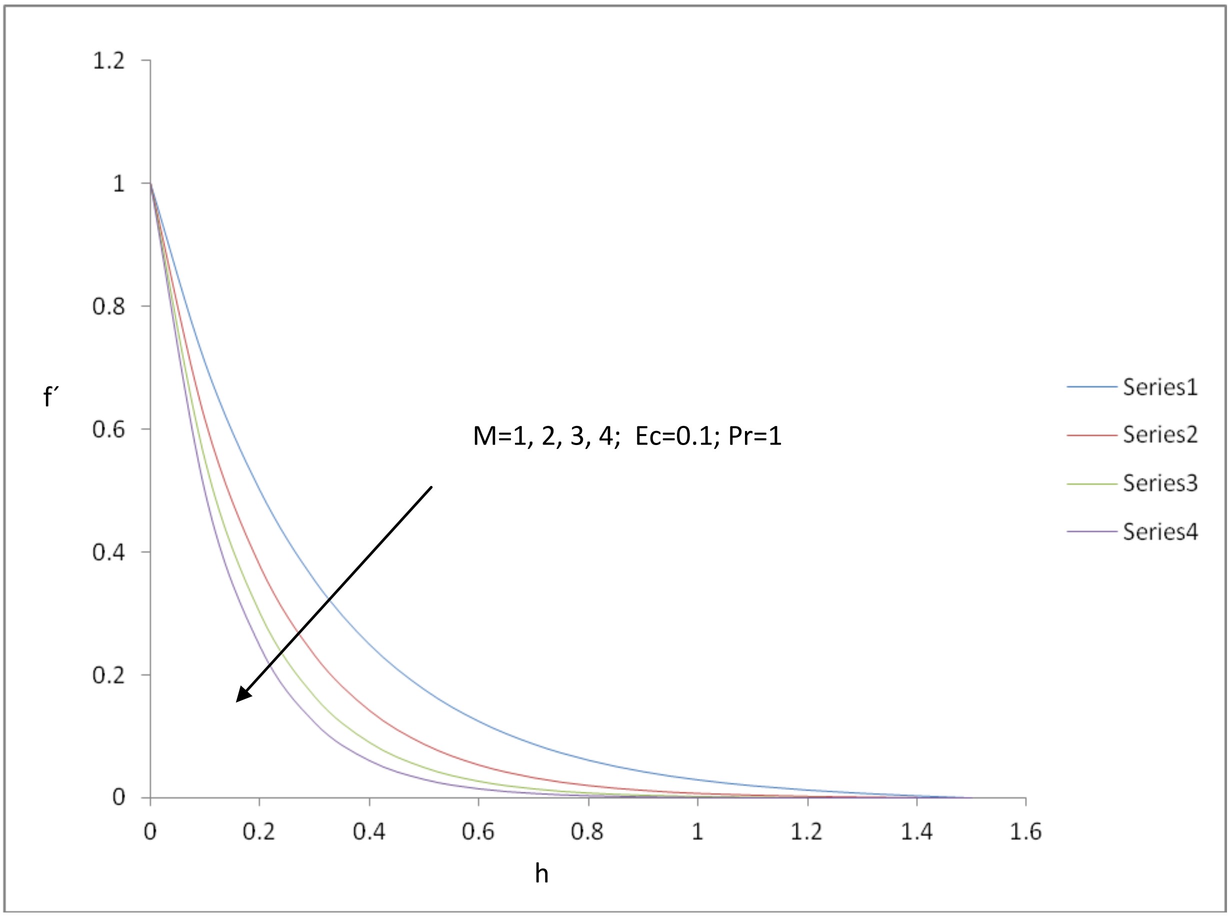


Fig1 Velocity profile for different values of M for Ec=0.1,Pr=1

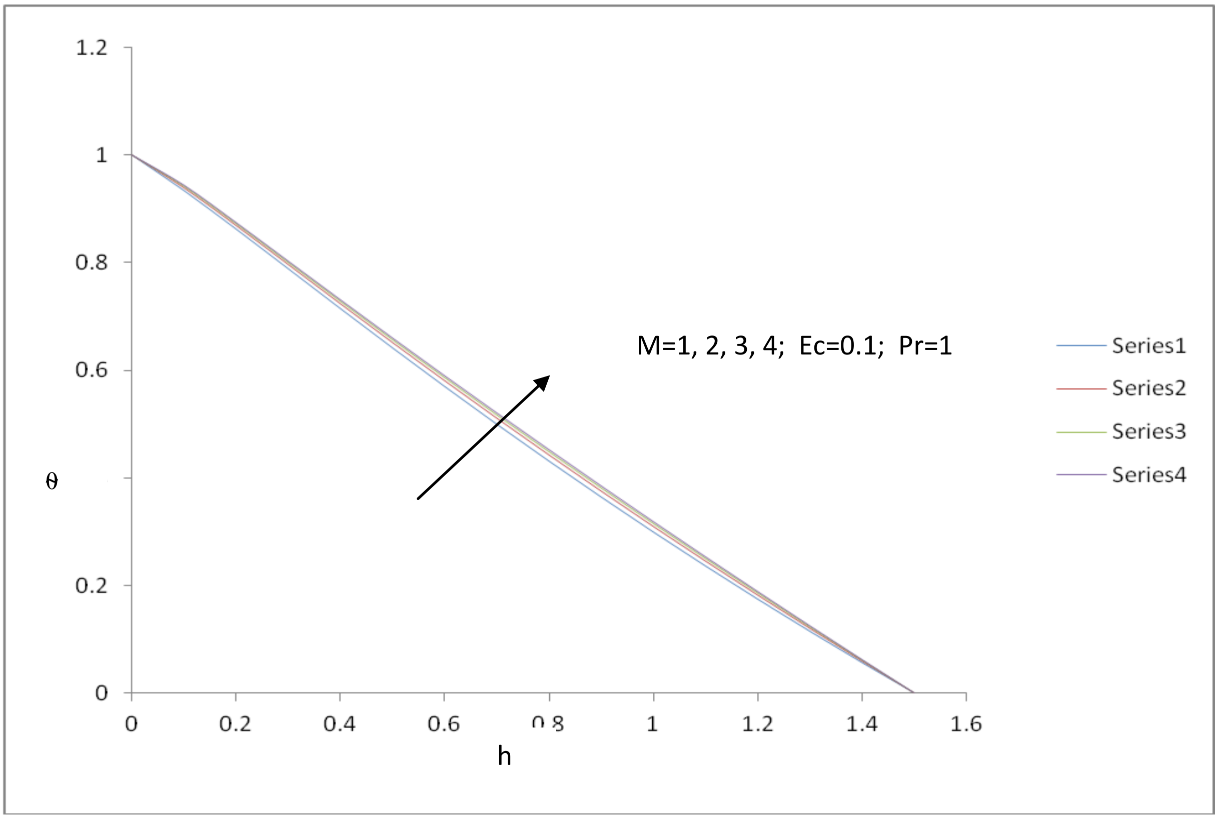


Fig 2 Temperature profile for different values of M for Ec=0.1,Pr=1

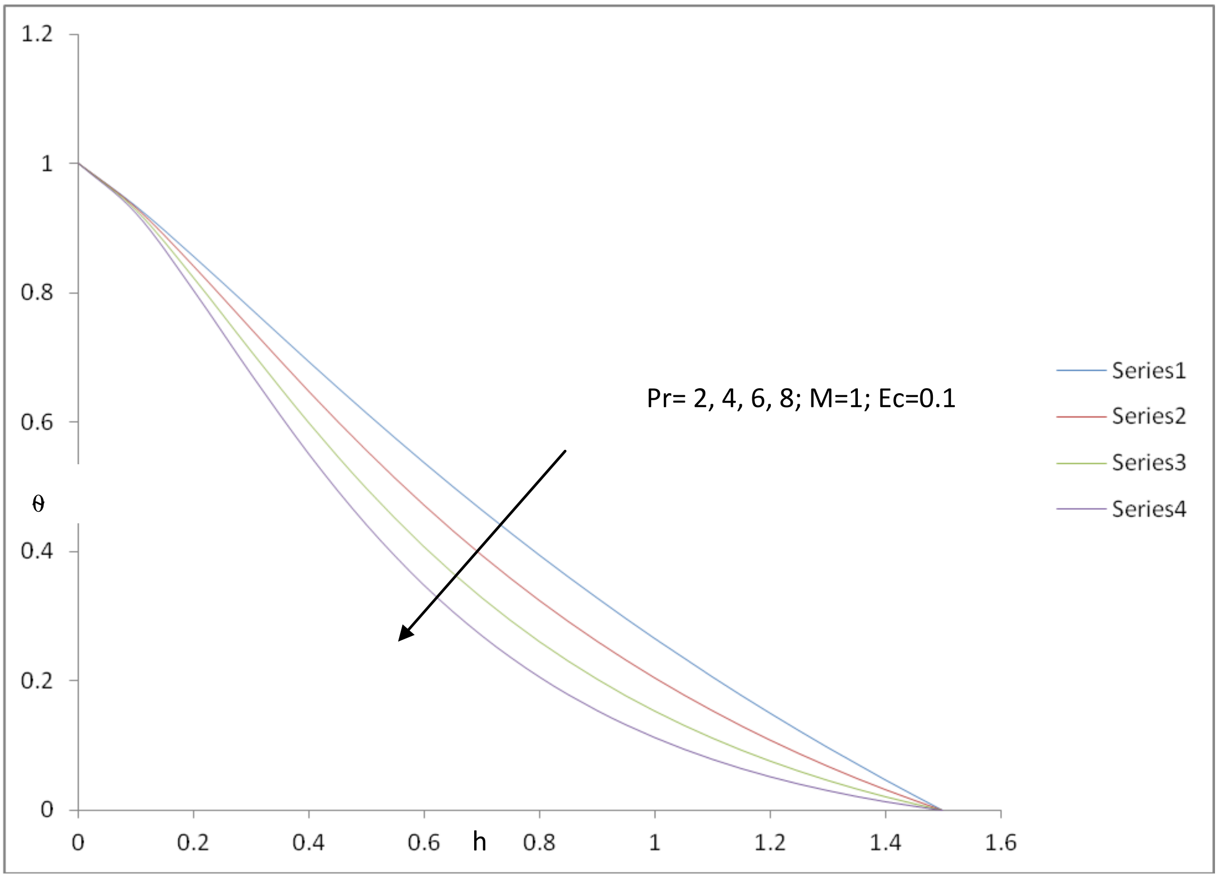


Fig 3 Temperature profile for different values of Pr for Ec=0.1,M=1

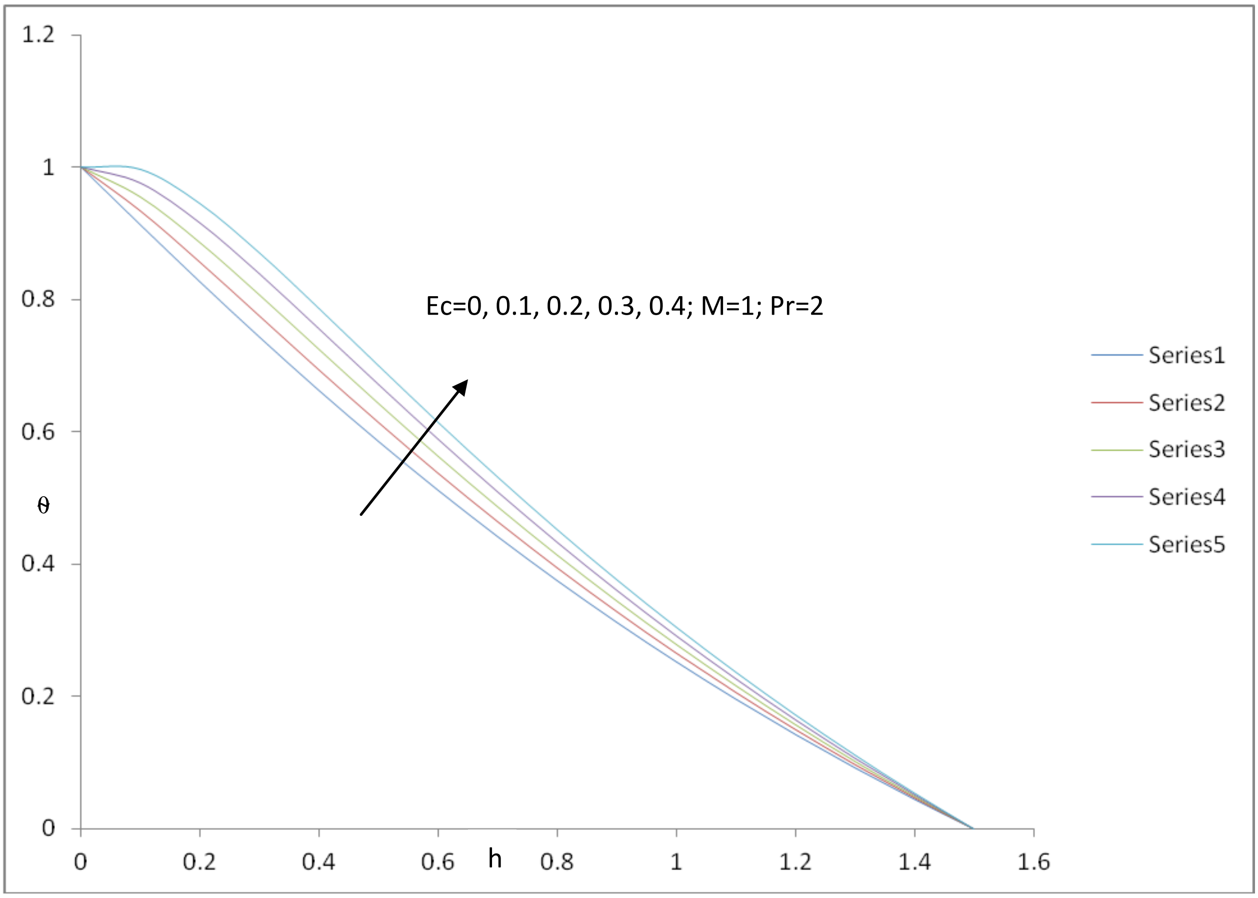


Fig 4 Temperature profile for different values of Ec for M=1,Pr=2

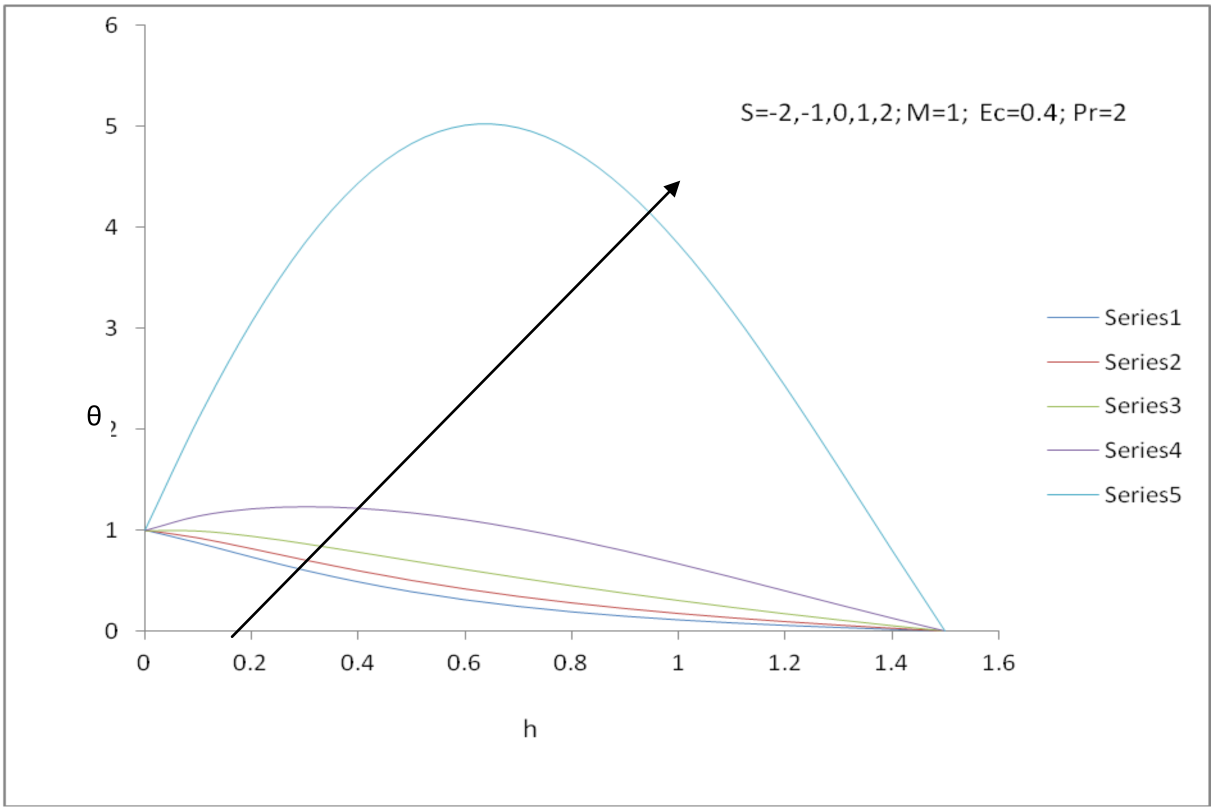


Fig 5 Temperature profile for different values of S for Ec=0.4, M=1,Pr=2

IV. RESULTS AND DISCUSSIONS:

Velocity profiles presented for various values of magnetic parameter M and given Pr are shown in fig.1. It is observed from the figure that the velocity decreases as magnetic field increases. It clearly indicates that the rate of transport is considerably reduced with the increase of M. And we note that the Prandtl number Pr has no influence on the velocity of fluid flow.

Temperature distributions presented for various values of Magnetic field parameter M, Prandtl number Pr and Eckert number Ec are shown in figures 2 to 4. The influence of Magnetic parameter M on temperature profiles for the fixed values of Pr and Ec is presented in fig. 2. It is evident from the figure that the temperature increases as the magnetic parameter increases. And this increase is very small.

Figure 3 shows the effect of Prandtl number Pr on the temperature profiles when M and Ec are constants. It is obvious from the figure that increase in the prandtl number decreases the temperature profiles which shows that the temperature in the boundary layer flow decreases.

Figure 4 reveals the effect of viscous dissipation on the temperature distribution while the M and Pr are fixed. It is seen from the figure that the temperature in the boundary layer flow increases with the increase in the Eckert number Ec.

Figure 5 reveals the effect of source term on the temperature distribution while the M, Ec and Pr are fixed. It is seen from the figure that the temperature in the boundary layer flow increases with the increase in the source term S.

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